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Estimation of daytime downward longwave radiation under clear and cloudy skies conditions over a sub-humid region

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Abstract Downward longwave radiation ($LW_{\downarrow}$) is a relevant variable for meteorological and climatic studies. Good estimates of this term are vitally important in correct determining of the net radiation, which, in turn, modulates the magnitude of the terms in the surface energy budget (e.g., evaporation). In remote sensing applications, the determination of daytime $LW_{\downarrow}$ is required for estimation of the net radiation using satellite data. $LW_{\downarrow}$ is not directly measured in weather stations and then is estimated using models with surface air temperature and humidity as input. In this paper, we identify the best models to estimate daytime downward longwave radiation from meteorological data in the sub-humid Pampean region. Several well-known models to estimate $LW_{\downarrow}$ under clear and cloudy skies were tested. We use downward radiation components and meteorological data registered at Tandil (Argentina) from 2006 to 2010 (840 days). In addition, we propose two multiple linear regression models (MLRM-1 and MLRM-2) to estimate $LW_{\downarrow}$ at the surface for all sky conditions. The new equations show better performance than the others models tested with root mean square errors between 12 and 16 W m^{-2}, bias close to zero and best agreements with measured data ($r^2 \geq 0.85$).

1 Introduction

Downward longwave radiation is a relevant variable for meteorological and climatic studies. Among them, energy balance studies at the Earth's surface depend critically on proper estimates of $LW_{\downarrow}$ (Duarte et al. 2006). Good estimates of this term are vitally important in correct determining of the net radiation, which, in turn, modulates the magnitude of the terms in the surface energy budget (e.g., evaporation). In remote sensing applications, the determination of daytime $LW_{\downarrow}$ is required for estimation of the net radiation using satellite data. For example, Bisht et al. (2005) estimated the net radiation with Moderate Resolution Imaging Spectroradiometer (MODIS) images in the Southern Great Plains, where the $LW_{\downarrow}$ was calculated using the near-surface air temperature and dew point temperature provided in the MODIS atmosphere profile product and equation proposed by Prata (1996). Furthermore, among other applications, the knowledge of $LW_{\downarrow}$ is required for the forecast of nocturnal frosts, fogs, temperature variation, and cloudiness; energy balance studies; the design of radiant cooling systems; as well as calculations on climate variability and global warming (Crawford and Duchon 1999; Gröbner et al. 2009).

Longwave radiation is emitted approximately in the range of 4.0–100.0 μm, mainly by H_2O, CO_2, and O_3 molecules and cloud water droplets (Idso and Jackson 1969). It can be measured directly by pyrgeometers; however, these measures are not usually done in weather stations (pyrgeometers are relatively expensive and sensitive when are compared to the pyranometers which measure shortwave radiation). Being difficult and expensive to measure directly, $LW_{\downarrow}$ is often estimated with radiation models based on more readily available data such as air temperature and humidity (Duarte et al. 2006).

When scattering is neglected (a good approximation in the longwave region especially in clear-sky conditions), the $LW_{\downarrow}$ at the surface is given by the following equation:

$$LW_{\downarrow} = -\int_0^{\infty}\int_{p_s}^{0} \pi B_{\lambda}[T(p)] \frac{\partial \tau_{\lambda;}(p_s, p)}{\partial p} \, d\, p\, d\, \lambda \quad (1)$$

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where p is the air pressure, p_s is the surface pressure, T is the temperature, λ is the wavelength, B_λ is the monochromatic Planck function, and τ_λ (p_s, p) is the monochromatic flux transmissivity from a pressure level, p, to the surface (Niemelä et al. 2001). To estimate $LW_\downarrow$ with Eq. (1), a parameterization of the emissivity together with the atmospheric profiles of temperature, humidity, and pressure is required.

If the relevant properties of the overlaying air column are known, the $LW_\downarrow$ at level ground can be calculated fairly accurately using radiative transfer codes (RTC). The RTC such as LOWTRAN (Kneizys et al. 1988), MODTRAN (Snell et al. 1995), SBDART (Ricchiazzi et al. 1998), and STREAMER (Key and Schweiger 1998) try to describe the actual emission and absorption processes in the atmosphere. Although they are admittedly more accurate, they also require more data (e.g., temperature and humidity profiles, cloud properties, and aerosols) that are not often available at the sites where $LW_\downarrow$ estimates are desired. Thus, lack of data tends to limit their applicability (Wright 1999; Duarte et al. 2006). In this sense, Viúdez-Mora et al. (2009) analyzed the performance of the SBDART using different approaches for the required atmospheric profiles. They showed a degradation of the results when the atmospheric radio soundings were not available.

Then, simple models (SM; which take into account variables meteorological measured near the surface level) to estimate the $LW_\downarrow$ are generally used. The SM implies some assumptions regarding the vertical structure of the atmosphere. In some cases, these assumptions are explicitly presented while in other cases they are implicitly considered through the fitting of local coefficients. From the viewpoint of thermal atmospheric irradiance, the atmosphere can be considered as a gray body with effective emissivity defined as $\varepsilon = LW_\downarrow/(\sigma\ T_a^4)$, being σ the Stefan–Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4}), T_a (in kelvin) the air temperature and $LW_\downarrow$ expressed in watts per square meter (Brutsaert 1984).

Under clear-sky conditions, ε can be modeled as a function of T_a, and/or vapor pressure, e_a, that are routinely measured in meteorological observatories and registered in automatic weather stations around the world (Alados et al. 2011). Then, Eq. (1) can be rewritten as:

$$LW_{\downarrow 0} = \varepsilon_0(T_a, e_a)\sigma T_a^4 \tag{2}$$

where subscript "0" indicates clear-sky conditions. There exist numerous formulations to estimate the $LW_{\downarrow 0}$ from simple meteorological data. Given the general scheme presented in Eq. (2), Ångström (1918) developed the first empirical relationship to estimate $LW_{\downarrow 0}$. Since then several authors have proposed different formulations, in this sense we can mention the equations presented by Brunt (1932), Swinbank (1963), Idso and Jackson (1969), Staley and Jurica (1972), Brutsaert (1975), Satterlund (1979), Idso (1981), Berdahl and Fromberg (1982), Culf and Gash (1993), Prata (1996), Dilley and O' Brien (1998), and Pérez-García (2004). Also, some authors have estimated the experimental coefficients to adapt the equations at different local conditions (e.g., Bilbao and de Miguel 2007; Lhomme et al. 2007; Choi et al. 2008; Alados et al. 2011; Marthews et al. 2011; among others).

On the other hand, under cloudy-sky conditions, the longwave radiation flux received at surface is substantially modified. The liquid water and ice absorb and emit longwave radiation more effectively than water in the vapor phase (most important atmospheric gas contributing to thermal radiation in the atmosphere) increasing the downward longwave radiation. It is the reason why the cloud cover plays an important role to estimate $LW_\downarrow$ (Lhomme et al. 2007). Then, a more general formulation to estimate $LW_\downarrow$ can be written as:

$$LW_\downarrow = \varepsilon_e(c, T_a, e_a)\sigma T_a^4 \tag{3}$$

where ε_e is the effective emissivity of the atmosphere (under all-sky conditions) and c (dimensionless) is the cloud fraction, being more difficult to estimate downward longwave radiation under cloudy-sky conditions. Generally, effective emissivity is defined as $\varepsilon_e = f(c).\varepsilon_0$ being $f(c)$ a function with c as input. These functions are called cloudy-sky correction models (CSCM). The c can be obtained from visual observations (Alados-Arboledas et al. 1995; Niemelä et al. 2001), from solar radiation measurements (Crawford and Duchon 1999) or occasionally from satellite data (Sugita and Brutsaert 1993). Also, whole-sky cameras can be used to estimate cloud fraction. Under cloudy-sky conditions, we can mention the formulas presented by Maykut and Church (1973), Jacobs (1978), Sugita and Brutsaert (1993), Konzelmann et al. (1994), Alados-Arboledas et al. (1995), Crawford and Duchon (1999), Sridhar and Elliott (2002), Iziomon et al. (2003), Duarte et al. (2006), Lhomme et al. (2007), among others.

In this study, we identify the best models or methodologies to estimate daytime downward longwave radiation from meteorological data in the sub-humid Pampean region (Tandil, Argentina). The specific goals of this study are: (a) assess the performance of well-known simple models with both original and local coefficients to estimate $LW_\downarrow$ under clear-sky conditions, (b) assess the performance the cloudy-sky correction models to estimate $LW_\downarrow$ under cloudy-sky conditions, and (c) evaluate to the performance of two new models using multiple linear regressions to estimate $LW_\downarrow$ under all-sky conditions with basic meteorological data as input.

2 Materials and methods

2.1 Study region and ground data

Tandil is located in the central-southeastern area of Buenos Aires province, Argentina, within the so-called Pampean region (Fig. 1). The climate is temperate and sub-humid, with warm summers and cool winters. The mean annual temperature is 14±1 °C, with a maximum monthly temperature of 22 °C in January and minimum of 8 °C in the colder months year (June, July, and August). The mornings are often cold, sometimes even in summer are quite fresh. There are often fogs in autumn and winter, and there are also abundant frosts in winter. Average annual rainfall is 900 mm (Tandil Station of the Argentinean National Meteorological Network, 37°14′S, 59°15′W, 175 m); the maximum monthly value is in March and the minimum is in August. Furthermore, average monthly values of wind speed, air relative humidity (RH), and solar radiation ($SW_{\downarrow}$) are 2.6±0.5 m s^{-1}, 83±7 %, and 190±80 W m^{-2}, respectively (Rivas and Caselles 2004; Carmona et al. 2012).

Data collected at eight measurement campaigns (between March 2007 and June 2010) were used in this study (Carmona et al. 2011). These campaigns were developed at different sites within 50 km radius—840 days in all. First campaign was conducted within the farm *Los Pilucos* (37°06′S, 59°07′W, 130 m) in March 2007. The following measurement campaigns were carried out in experimental plots of the Universidad Nacional del Centro de la Provincia de Buenos Aires (UNCPBA, 37°19′S, 59°05′W, 214 m) between July 2007 and December 2009, and the last two campaigns were carried out within the farm *Laura Leofú* (37°14′S, 59°34′W, 235 m) between December 2009 and June 2010 (Table 1).

Downward longwave radiation was measured with a CG3 pyrgeometer of spectral range comprised between 5 and 50 μm. Solar radiation was measured with a CM3 pyranometer (spectral range 0.305–2.800 μm). CG3 and CM3 sensors are part of a four-component CNR1 radiometer (Kipp and Zonen). The radiometer calibration was provided by manufacturer. The sensitivity of the radiometer ranges from 10 to 35 μm/W m^{-2}, and the sensors (CG3 and CM3) have a directional error less than or equal to 25 W m^{-2}. CNR1 radiometer was visually inspected during campaigns and was recalibration bi-annually. Air temperature/relative humidity was also measured with a CS215-L16 probe (Campbell Scientific Inc.). The error of the temperature sensor is 0.4 °C (between 5 and 40 °C). The error associated

Fig. 1 Experimental region location

Table 1 Experimental sites with its land uses, number of data days, and year of the campaigns

Site	Land use	Count of days	Year
Los Pilucos	Soybean	28	2007
UNCPBA	Pasture	168	2007–2008
UNCPBA	Oats	83	2008
UNCPBA	Pasture	276	2008–2009
UNCPBA	Oats	75	2009
UNCPBA	Pasture	40	2009
Laura Leofú	Soybean	107	2010
Laura Leofú	Bare soil	63	2010

with the relative humidity measurement is ±2 % over 10–90 % and ±4 % over 0–100 %. All sensors were installed at about 2 m above the ground. Data were acquired by a CR10X data-logger (Campbell Scientific Inc.), and 15-min averages were recorded on a storage module for later processing. Station was continuously powered by a 12-V battery connected to a 20-W solar panel.

All data recorded were processed for their use. Furthermore, e_a and c were needed. The e_a (in hectopascal) was obtained as (Allen et al. 1998)

$$e_a = e_s\left(\frac{HR}{100}\right) = \left(6.108\exp\left[\frac{17.27T_a}{T_a + 237.3}\right]\right)\left(\frac{HR}{100}\right) \quad (4)$$

where e_s (in hectopascal) is the saturation vapor pressure. T_a and RH are expressed in degree Celsius and percentage, respectively. As cloud fraction measurements were not available, it was estimated by (Crawford and Duchon 1999):

$$c = (1-s) = 1 - \frac{SW_{\downarrow}}{SW_{\downarrow 0}} \quad (5)$$

where s (dimensionless) is defined as the ratio between the measured incoming solar radiation, $SW_{\downarrow}$, and the theoretical incoming clear-sky solar radiation, $SW_{\downarrow 0}$ (value calculated, see the "Appendix"). Nighttime measurements were excluded because the ratio s cannot be calculated. Furthermore, measurements when $SW_{\downarrow 0} \leq 100$ W m^{-2} were excluded because the cloud fraction estimate errors can be significant. Thus, a dataset of 8,393 daytime measurements was available for the study. Figure 2 shows hourly values of $SW_{\downarrow}$ (in watts per square meter), $LW_{\downarrow}$ (in watts per square meter), T_a (in kelvin), and RH (in percent) that were used. Basic statistics of those data are summarized in Table 2.

2.2 Clear-sky downward longwave radiation

Six previously published SM to estimate downward longwave radiation for clear sky were evaluated. First the models with original coefficients were tested and then with experimental coefficients which were estimated at local conditions. We identified the measures as clear-sky conditions as those which have cloud fraction values calculated less than or equal to 0.05 (Duarte et al. 2006). The SM are presented as follows with its original coefficients:

1. The empirical formula developed by Brunt (1932) expressing $LW_{\downarrow 0}$ in terms of an effective emissivity computed from vapor pressure measured at the screen level. This formula has two empirical coefficients that can be obtained from local observational data. Brunt's (1932) equation is expressed as:

$$LW_{\downarrow 0} = \left(a_1 + b_1 e_a^{1/2}\right)\sigma T_a^4 \quad (6)$$

with a_1=0.55 and b_1=0.065 hPa$^{-1/2}$ according original work in Benson (UK). These coefficients vary somewhat significantly from location to location. In general, b_1 varies more than a_1, with the former showing a variability of about 32 % and the latter only 13 % according to Iziomon et al. (2003).

2. Swinbank (1963) developed an equation for estimating $LW_{\downarrow 0}$ using data from Australia, the Indian Ocean at low latitudes, England, and France and for T_a in the range 2–29 °C that only depended on screen level air temperature. It is given by the expression:

$$LW_{\downarrow 0} = \left(a_2 T_a^2\right)\sigma T_a^4 \quad (7)$$

being a_2=9.36×10^{-6} K^{-2}.

3. Idso and Jackson (1969) using data from Point Barrow (AK, USA), Phoenix (AZ, USA), Aspendale (VIC, Australia), Kerang (VIC, Australia), and the Indian Ocean and for T_a in the range from −29 to 37 °C and proposed the equation:

$$LW_{\downarrow 0} = \left(1 - a_3\exp\left[b_3(273 - T_a)^2\right]\right)\sigma T_a^4 \quad (8)$$

with a_3=0.261 and b_3=−7.77×10^{-4} K^{-2}. In these last two equations, the effective emissivity of the atmosphere is a function of T_a only.

4. The Brutsaert (1975) model is based on analytical equations using radiative transfer theory and data from several other authors. It is a function of vapor pressure and temperature at screen level. It is given by the following expression:

$$LW_{\downarrow 0} = \left(a_4 (e_a/T_a)^{b_4}\right)\sigma T_a^4 \quad (9)$$

with a_4=1.24 (K/hPa)^{b4} and b_4=1/7.

5. Idso (1981) derived an equation based on observations at Phoenix (AZ, USA) with T_a data in the range from −10 to 45 °C which includes both T_a (in kelvin) and e_a (hectopascal). It is given by the expression:

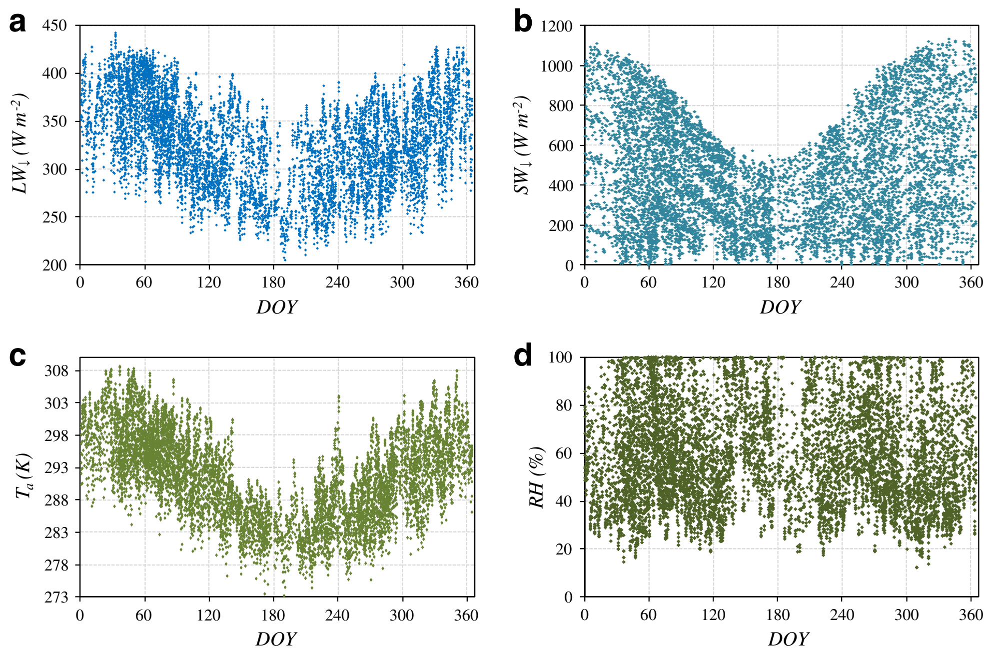

Fig. 2 Hourly averages values of **a** downward longwave radiation, **b** solar radiation, **c** air temperature, and **d** relative humidity (*DOY* day of year)

$$\mathrm{LW}_{\downarrow 0} = (a_5 + b_5 \mathrm{e_a} \exp[1,500/T_\mathrm{a}])\sigma T_\mathrm{a}^4 \qquad (10)$$

with a_5=0.7 and b_5=5.95×10^{-5} hPa^{-1}.

6. Prata (1996) presented an equation which basically follows Brutsaert (1975) derivation using adjusted slab emissivity. The formula was extensively tested using longwave measurements covering a large range of environmental temperatures (−40 to 40 °C) and by using radiosonde profiles and an accurate radiative transfer code (LOWTRAN-7). It is given by the following expression:

$$\mathrm{LW}_{\downarrow 0} = \left(1 - [(1+\mathrm{w})\exp\left(-(a_6 + b_6\mathrm{w})^{1/2}\right)]\right)\sigma T_\mathrm{a}^4 \qquad (11)$$

with a_6=1.2, b_6=3 g^{-1} cm^2, and w is the precipitable water content calculated as 46.5(e_a/T_a)g cm^{-2}.

Both physical and empirical model parameters and performance are significantly affected by geographical location and local atmospheric conditions and require site specific validation and parameterization (Choi et al. 2008).

Table 2 Statistical summary of downward longwave radiation, solar radiation, air temperature, and relative humidity

Statistical	$LW_{\downarrow}$ (W m^{-2})	$SW_{\downarrow}$ (W m^{-2})	T_a (K)	RH (%)
Observed mean	330	460	292	59
Standard deviation	50	270	280	22
Minimum	205	0	273	12
Maximum	440	1,140	309	100

Tandil dataset for the period 2006–2010 (8,393 daytime hourly values)

2.3 Cloudy-sky downward longwave radiation

Table 3 shows six cloudy-sky correction models with its original coefficients which were tested in this study. These equations try to estimate the increase in downward longwave radiation produced by clouds. First four equations shown were originally tested with c estimated by human observers and last two with c estimated by Eq. (5) (Crawford and Duchon 1999).

In theory, $f(c)$=($\varepsilon_e/\varepsilon_0$)=($LW_{\downarrow}/LW_{\downarrow 0}$) should be equal to 1 for c=0 (cloud-free conditions), which is not the case of the last cloudy-sky correction models (CSCM) showed in Table 3 since $f(c)$=1.03. Lhomme et al. (2007) justified this small discrepancy as that is due to the purely statistical character of the relationship.

Also, according to the methodology followed by Duarte et al. (2006) the general forms of Eqs. (12)–(17) were adjusted at local conditions. The general forms are given by equations:

Table 3 Cloudy-sky correction models evaluated in this study

Source	Cloudy-sky correction models	
Maykut and Church (1973)	$\mathrm{LW}_{\downarrow} = \mathrm{LW}_{\downarrow 0}(1 + 0.22\mathrm{c}^{2.75})$	(12)
Jacobs (1978)	$\mathrm{LW}_{\downarrow} = \mathrm{LW}_{\downarrow 0}(1 + 0.26\mathrm{c})$	(13)
Suguita and Brutsaert (1993)	$\mathrm{LW}_{\downarrow} = \mathrm{LW}_{\downarrow 0}(1 + 0.0496\mathrm{c}^{2.45})$	(14)
Konzelmann et al. (1994)	$\mathrm{LW}_{\downarrow} = \mathrm{LW}_{\downarrow 0}(1 - \mathrm{c}^4) + 0.952\mathrm{c}^4\sigma \mathrm{T}_\mathrm{a}^4$	(15)
Crawford and Duchon (1999)	$\mathrm{LW}_{\downarrow} = \mathrm{LW}_{\downarrow 0}(1 - \mathrm{c}) + \mathrm{c}\sigma \mathrm{T}_\mathrm{a}^4$	(16)
Lhomme et al. (2007)	$\mathrm{LW}_{\downarrow} = \mathrm{LW}_{\downarrow 0}(1.03 + 0.34\mathrm{c})$	(17)

$$\text{CSCM}-1 \quad \text{LW}_{\downarrow} = \text{LW}_{\downarrow 0}\left(1+\alpha \text{c}^{\beta}\right) \tag{18}$$

$$\text{CSCM}-2 \quad \text{LW}_{\downarrow} = \text{LW}_{\downarrow 0}(1-\text{c}^{\mu}) + \gamma \text{c}^{\mu}\sigma \text{T}_{\text{a}}^{4} \tag{19}$$

where α, β, μ, and γ are experimental coefficients which in general depend on cloud characteristics.

2.4 Proposed models under all-sky conditions

We proposed two multiple linear regression models (MLRM) to estimate downward longwave radiation under clear- and cloudy-sky conditions. The first proposed model, MLRM-1, is expressed as:

$$\text{LW}_{\downarrow} = \varepsilon_{\text{e}}\sigma \text{T}_{\text{a}}^{4} = \varepsilon_{0}\text{f}(\text{c})\sigma \text{T}_{\text{a}}^{4} = [\alpha_0 + \alpha_1 \text{T}_{\text{a}} + \alpha_2 \text{RH}] f(c)\sigma T_{\text{a}}^{4} \tag{20}$$

where α_0, α_1, and α_2 are local coefficients and $f(c)$ is the better performance cloudy-sky correction model (between Eqs. (12)–(19)).

The second proposed model, MLRM-2, is expressed as:

$$\text{LW}_{\downarrow} = \varepsilon_{\text{e}}\sigma T_{\text{a}}^{4} = [\beta_0 + \beta_1 T_{\text{a}} + \beta_2 \text{RH} + \beta_3 \text{c}]\sigma T_{\text{a}}^{4} \tag{21}$$

where β_0, β_1, β_2, and β_3 are local coefficients. In the two previous equations, T_a and RH are expressed in kelvin and percentage, respectively.

To evaluate the performance of all models, following statistics were considered: the mean bias error (BIAS), the root mean square error (RMSE), the percent root mean square error ($\text{PRMSE} = 100\left[\text{RMSE}/\overline{\text{LW}_{\downarrow}}\right]$, where $\overline{\text{LW}_{\downarrow}}$ the means of the observed values) and the determination coefficient (r^2) with the intercept (a) and slope (b) of the linear regression. The slope of the linear regression forced through the origin (b*) also was considered because provides information about the relative underestimation or overestimation associated with the models.

3 Results and discussion

3.1 Clear-sky conditions

First, we selected a sample of 3,443 hourly data for which average cover fraction calculated from Eq. (5) was less than or equal to 0.05. It was randomly divided into two subsets: 1/3 sample (N=1,185) to test the clear-sky models and the remaining 2/3 sample (N=2,258) to fitting its coefficients at local conditions.

Figure 3 shows plots of hourly estimates of $\text{LW}_{\downarrow 0}$ from six considered models versus measured values (CG3 sensor). The statistics results of the tested models are summarized in Table 4.

Results show that the six equations with its original coefficients overestimated the measured $\text{LW}_{\downarrow 0}$. Swinbank (1963) and Idso and Jackson (1969) equations presented the highest errors as compared to the other models. These models presented highest BIAS (~30 W m^{-2}) and RMSE (~40 W m^{-2}) and worse agreement with measured data (r^2=0.78). The best results were obtained with Brunt (1932) and Brutsaert (1975) equations, with the smallest values of BIAS, RMSE, and "a", b* closer to 1 and good agreement between estimated and observed values with r^2=0.89. Furthermore, we compare our results with those reported by other authors that used similar methodology (Table 5).

Table 5 shows statistical results of studies that have been developed under several weather conditions but none of them under subhumid temperate climate such as presented here. In general, it can observe that Brunt (1932), Brutsaert (1975), and Prata (1996) equations show better performances, and it is regardless of the weather conditions of each region. Also, results of other authors confirmed that generally simple models overestimate the daytime measured $\text{LW}_{\downarrow 0}$.

Since some of these models were derived for nighttime data and other models from to daily data, it is logical to find performance differences when used daytime hourly data. To improve the daytime performance of the models, we adjusted its empirical coefficients at local conditions.

As mentioned above, to do that we used the 2/3 remaining sample for clear-sky conditions. The local coefficients were obtained using a nonlinear least squares fit to adjust the values in the iterative procedure. The original and local coefficient values for the six considered models are shown in Table 6.

For Brunt (1932) model, the b_1 coefficient value was significantly different to original value while that a_1 was similar. The largest difference found for b_1 is in line with the analyzed by Iziomon et al. (2003). These investigators presented the a_1 and b_1 coefficients obtained from measures on their study sites as well as those values reported by others. They showed which these coefficients vary from location to location and which b_1 has a largest dependence with values range of e_a (see Table 4 of Iziomon et al. (2003)). The calibrated coefficients present percentage differences of about 10 % respect to original values for Swinbank (1963), Brutsaert (1975), and Idso (1981) equations. Furthermore, the largest differences were shown for Idso and Jackson (1969) and Prata (1996) coefficients. In particular, Alados et al. (2011) with Tabernas dataset registrated at Almeria (Spain, semiarid climate) found a very different value of b_3 (-1.30×10^{-4} K^{-2}) and a similar a_3 value (0.258) which agrees with our results.

From the data subset of N=1,185 (1/3 sample) we tested clear-sky equations using the calibrated coefficients at local conditions (Table 6). In this case, Fig. 4 shows hourly estimates of $\text{LW}_{\downarrow 0}$ versus measured values, and the statistic

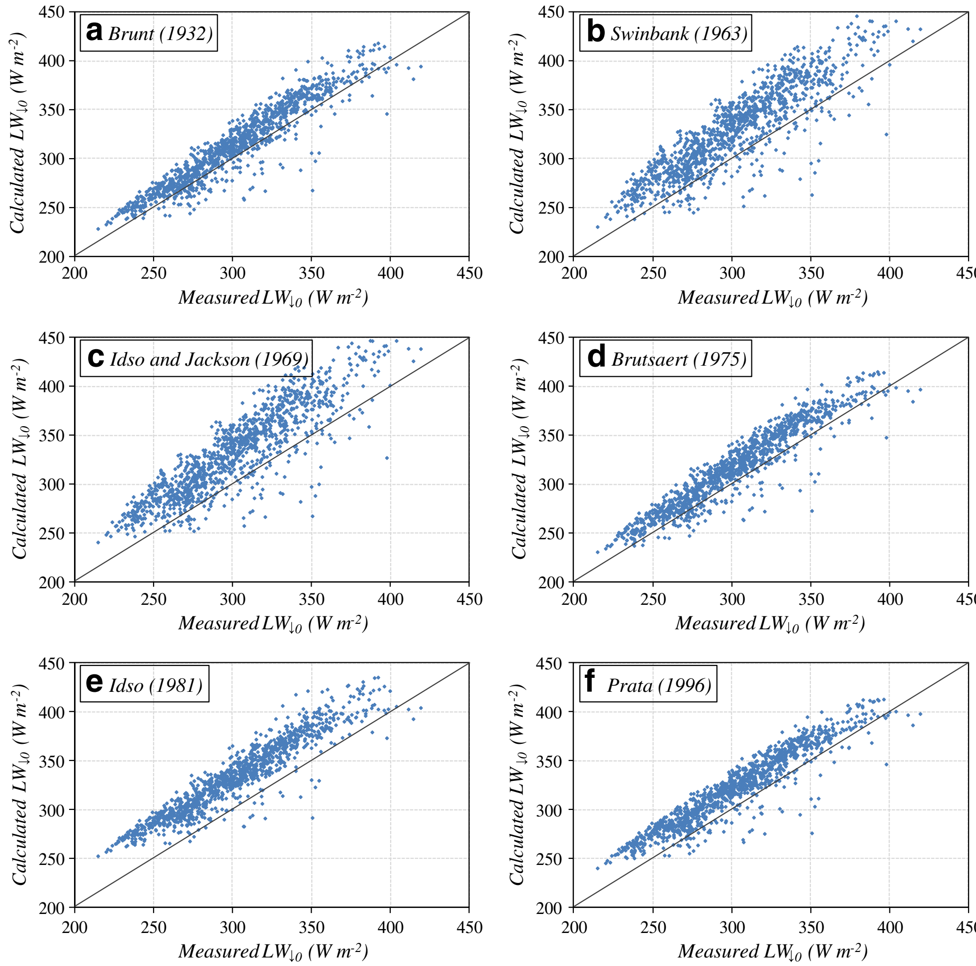

Fig. 3 Comparison of clear-sky downward longwave radiation results for **a** Brunt, **b** Swinbank, **c** Idso and Jackson, **d** Brutsaert, **e** Idso, and **f** Prata models using its original coefficients. The 1:1 line is shown on each plot

results of the validation of adjusted models are summarized in Table 7.

Results show that the locally calibrated formulas provide better results than those with original coefficients. These can be divided into two groups: (1) those which only have T_a and (2) those equations also with e_a as input to estimate $LW_{\downarrow 0}$. Those models which do not employ the air relative humidity (Swinbank (1963) and Idso and Jackson (1969)), to have a measure of the humidity effect on atmospheric path length, show worse agreements with measured data (r^2=0.78) and larger errors with PRMSE equal to 6 %.

On other hand, the second group—Brunt (1932), Brutsaert (1975), Idso (1981), and Prata (1996) equations—show best results with RMSE= 13 W m^{-2} and highest r^2 (~0.9). Within this group are not significant differences between models performance. Only a small difference can be observed between Brunt (1932), Brutsaert (1975), and Prata (1996) models respect to Idso (1981) equations if we analyze statistics of the linear regression. We confirmed that these differences are not significant with an ANOVA test.

Given the results shown in Table 7, models can be ranked according to their performance: (1) Brutsaert (1975), Brunt

Table 4 Summary statistics of simple models using its original coefficients

Simple models	BIAS (W m^{-2})	RMSE (W m^{-2})	PRMSE (%)	a (W m^{-2})	b	r^2	b^*
Brunt (1932)	12	18	6	10±3	1.01±0.01	0.89	1.039±0.001
Swinbank (1963)	30	40	13	22±5	1.02±0.02	0.78	1.092±0.002
Idso and Jackson (1969)	30	40	13	22±5	1.03±0.02	0.78	1.102±0.002
Brutsaert (1975)	15	20	7	14±3	1.00±0.01	0.89	1.049±0.001
Idso (1981)	30	30	10	50±3	0.94±0.01	0.88	1.102±0.001
Prata (1996)	20	23	8	40±3	0.92±0.01	0.89	1.063±0.001

Table 5 Statistical results for clear-sky conditions obtained for others authors using its original coefficients

Author/s	MBE (W m^{-2})	RMSE (W m^{-2})	PRMSE (%)	*a* (W m^{-2})	*b*	r^2	*b**
Brunt (1932)							
Iziomon et al. (2003)	−19/20	40/30	11/12	–	–	–	–
Bilbao and de Miguel (2007)	−3	11	–	–	–	–	–
Lhomme et al. (2007)	29	32	–	–	–	–	–
Choi et al. (2008)	4	12	4	–	–	0.87	1.014
Alados et al. (2011)	−8	15	5	–	–	0.88	0.975
Swinbank (1963)							
Iziomon et al. (2003)	−23/26	40/40	12/15	–	–	–	–
Duarte et al. (2006)	24	29	9	1.133	−17	0.91	–
Bilbao and de Miguel (2007)	32	40	–	–	–	–	–
Lhomme et al. (2007)	59	64	–	–	–	–	–
Idso and Jackson (1969)							
Iziomon et al. (2003)	−13/3	40/50	11/19	–	–	–	–
Duarte et al. (2006)	27	31	10	1.141	−17	0.91	–
Choi et al. (2008)	10	26	9	–	–	0.84	1.037
Alados et al. (2011)	25	35	11	–	–	0.77	1.081
Brutsaert (1975)							
Iziomon et al. (2003)	−19/20	30/30	10/12	–	–	0.78/0.55	–
Duarte et al. (2006)	13	15	5	0.903	43	0.96	–
Bilbao and de Miguel (2007)	16	20	–	–	–	–	–
Lhomme et al. (2007)	12	15	–	–	–	–	–
Choi et al. (2008)	8	14	5	–	–	0.87	1.029
Alados et al. (2011)	4	15	5	–	–	0.89	1.013
Idso (1981)							
Duarte et al. (2006)	29	31	10	0.830	81	0.96	–
Bilbao and de Miguel (2007)	30	32	–	–	–	–	–
Lhomme et al. (2007)	47	50	–	–	–	–	–
Prata (1996)							
Duarte et al. (2006)	17	19	6	0.869	57	0.96	–
Lhomme et al. (2007)	37	40	–	–	–	–	–
Choi et al. (2008)	17	16	–	–	–	0.87	1.041
Alados et al. (2011)	9	16	5	–	–	0.89	1.013
	Sites				Period		
Iziomon et al. (2003)	Bremgarten (lowland)/Feldberg (Mountain)—Germany				1991–1996		
Duarte et al. (2006)	Ponta Grossa (humid region, subtropical climate)—Brazil				2003–2004		
Bilbao and de Miguel (2007)	Valladolid (Mediterranean continental climate)—Spain				2001–2004		
Lhomme et al. (2007)	Condori (Andean Altiplano)—Bolivia				2005		
Choi et al. (2008)	Florida (humid region, subtropical climate)—USA				2004–2005		
Alados et al. (2011)	Tabernas, Almeria (semiarid climate)—Spain				2002		

(1932), and Prata (1996) equations, (4) Idso (1981) equation, (5) Swinbank (1963) equation, and (6) Idso and Jackson (1969) equation. We choose the Brutsaert's equation (with its calibrated coefficients) to estimate $LW_{\downarrow 0}$ for next section of this study, where evaluates several models under cloudy-sky conditions.

3.2 Cloudy-sky conditions

Calculations of $LW_{\downarrow}$ for cloudy sky were conducted using those measures which have cloud fraction values estimated greater than to 0.05. With similar methodology of the used for clear-sky conditions, the data were randomly divided into two subsets:

Table 6 Comparison of original and local coefficient values for the six clear-sky models

Author/s	Coefficients	Original values	Adjusted values	% difference
Brunt (1932)	a_1	0.55	0.556±0.004	1
	b_1 (hPa$^{-1/2}$)	0.065	0.054±0.001	−17
Swinbank (1963)	a_2 (K^{-2})	9.36×10^{-6}	$(8.55\pm0.01)\times10^{-6}$	−9
Idso and Jackson (1969)	a_3	0.261	0.303±0.002	16
	b_3 (K^{-2})	-7.77×10^{-4}	$(-2.8\pm0.2)\times10^{-4}$	−64
Brutsaert (1975)	a_4 (K^{b4}hPa^{-b4})	1.24	1.11±0.01	−10
	b_4	0.143	0.123±0.003	−14
Idso (1981)	a_5	0.7	0.630±0.002	−10
	b_5 (hPa^{-1})	5.95×10^{-5}	$(5.5\pm0.1)\times10^{-5}$	−8
Prata (1996)	a_6	1.2	0.79±0.03	−34
	b_6 (g^{-1} cm^2)	3	2.68±0.02	−11

1/3 sample (N=1,613) to test the cloudy-sky models and the remaining 2/3 sample (N=3,337) to calibrate coefficients of the general forms (Eqs. (18) and (19)) at local conditions.

Generally, in several studies, these calculations are conducted using all data (also when $c<0.05$) but we decide not to include the data in clear-sky conditions not to mask the performance of each cloudy-sky correction model tested. Figure 5 shows plots of hourly estimates of $LW_\downarrow$ versus measured values. Furthermore, results of the general forms are shown. The statistics results are summarized in Table 8.

Also we present the results obtained with Brutsaert's equation (with its calibrated coefficients) without CSCM to estimate $LW_\downarrow$. In this case, larger errors are observed, being the BIAS value equal to −40 W m^{-2}. This indicates that there is some underestimation because the cloud effects that tend to increase $LW_\downarrow$ are not accounted appropriately.

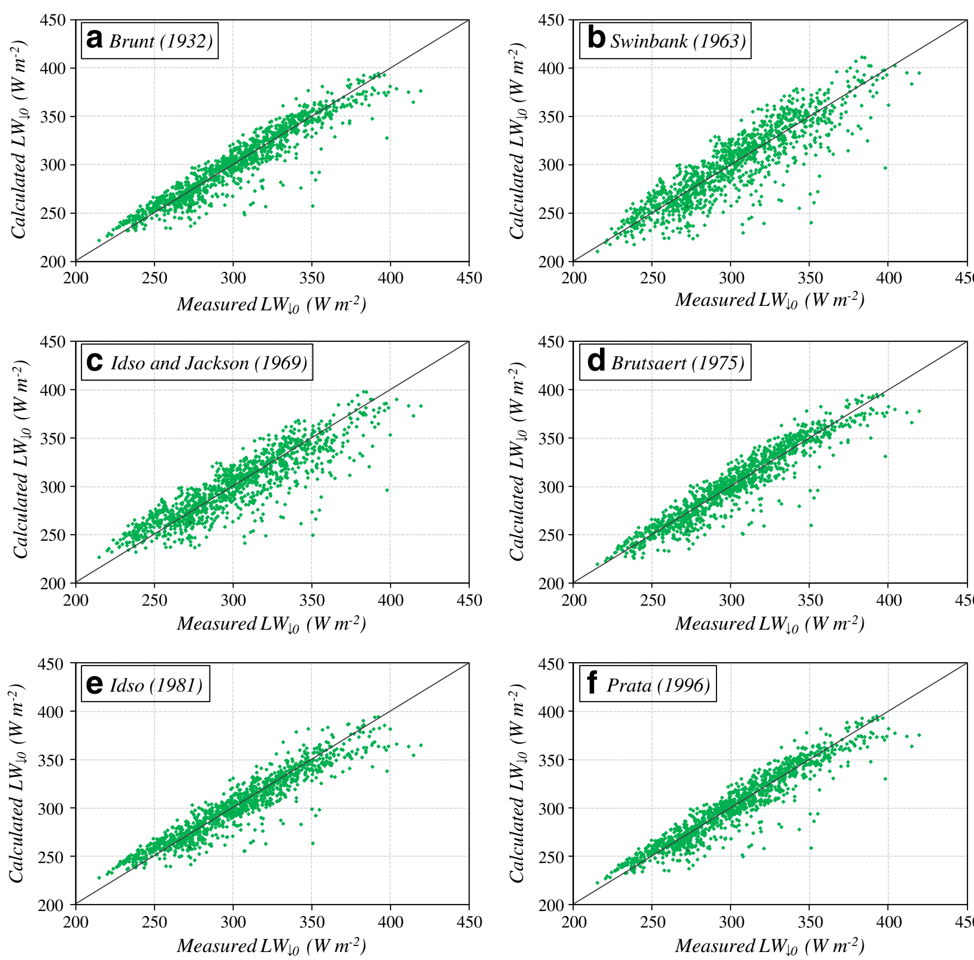

Fig. 4 Comparison of clear-sky downward longwave radiation results for **a** Brunt, **b** Swinbank, **c** Idso and Jackson, **d** Brutsaert, **e** Idso, and **f** Prata models using its adjusted coefficients at local conditions. The 1:1 line is shown on each plot

Table 7 Summary statistics of simple models using the adjusted coefficients

Simple models	BIAS (W m^{-2})	RMSE (W m^{-2})	PRMSE (%)	a (W m^{-2})	b	r^2	b*
Brunt (1932)	0	13	4	24±3	0.92±0.01	0.89	0.998±0.001
Swinbank (1963)	0	19	6	21±4	0.83±0.02	0.78	0.998±0.002
Idso and Jackson (1969)	−1	18	6	60±4	0.80±0.01	0.78	0.994±0.002
Brutsaert (1975)	0	13	4	27±3	0.91±0.01	0.89	0.998±0.001
Idso (1981)	−1	13	4	40±3	0.86±0.01	0.88	0.996±0.001
Prata (1996)	0	13	4	25±3	0.92±0.01	0.89	0.998±0.001

The models attempt to consider this distortion with c as input variable; in this way, those developed CSCM by Maykut and Church (1973), Suguita and Brutsaert (1993), and Konzelmann et al. (1994) presented worst performers. Best results were obtained with Crawford and Duchon (1999) model, which presents no bias, RMSE is equal to 17 W m^{-2}, and has very good agreement with measured data (r^2=0.83).

The coefficients α, β, μ, and γ of the general forms were fitted with measured data (N=3,337). The calibrated equations are expressed as:

$$\mathrm{CSCM}-1: \quad \mathrm{LW}_{\downarrow} = \mathrm{LW}_{\downarrow 0}\left(1+0.283c^{0.78}\right) \tag{22}$$

and

$$\mathrm{CSCM}-2: \quad \mathrm{LW}_{\downarrow} = \mathrm{LW}_{\downarrow 0}\left(1-c^{0.85}\right)+0.989\mathrm{c}^{0.85}\sigma T_{\mathrm{a}}^{4} \tag{23}$$

being α=0.283±0.003, β=0.78±0.02, μ=0.85±0.01, and γ=0.989±0.002.

These adjusted general forms showed similar or worse performances that the Crawford and Duchon (1999) model. Duarte et al. (2006), Choi et al. (2008), and Alados et al. (2011) among others have also shown in their investigations the good performance of this model to estimate $\mathrm{LW}_{\downarrow}$ considering the additional longwave radiation flux by clouds.

Duarte et al. (2006) used information measured at Ponta Grossa (25°08′S, 50°04′W, 890 m, Brazil), between 2003 and 2004 (279 days), a humid region with subtropical climate. In their study, Crawford and Duchon (1999) model showed best results with RMSE=22 W m^{-2} and BIAS close to zero (−9 W m^{-2}).

Choi et al. (2008) used data collected at central Florida by 11 net radiation experiment sites (two open water, two wetland, two urban, two rangeland, one forest, and two agriculture sites) from January 1, 2004 to December 31, 2005. The central Florida study region has a humid, subtropical climate, with an average annual rainfall of 1,500 mm. Almost 70 % of the annual rainfall occurs from May to November. Average annual temperature is 32.2 °C, and average annual relative humidity is higher than 50 %. They recommended to use the Crawford and Duchon (1999) equation that provide reasonable estimates with relatively high accuracy and low errors under typical convective cloud conditions in Florida, being RMSE values between 10 and 18 W m^{-2}, BIAS close to zero and r^2~0.90.

Alados et al. (2011) used measurements measured at Taberna (37°8′N, 2°22′W, 630 m, Almería, Spain) and Palaiseau (48°43′N, 2°13′E, 156 m, France). The Tabernas dataset was registered at the Rambla Honda field site, Almeria, Spain from 2001 to 2003. Tabernas is partially surrounded by the Betic cordillera and lee of the Sierra de los Filabres, Sierra Nevada, and Sierra of Gádor. The climate is semiarid with a mean annual temperature of 16 °C; mean rainfall is 279 mm which falls mainly in winter, followed by a dry period centered on the months of June–September. The station of Palaiseau is located 25 km to the West of Paris (France). The site is a semi-urban environment divided equally in agricultural fields, wooded areas, and housing and industrial developments. The prevailing winds are westerlies, blowing air of maritime origin over the site. Northeasterly winds occur quite frequently, as well advecting polluted air from the Paris metropolitan area over the site. Their results showed that Crawford and Duchon (1999) model presents similar performances in both experimental sites, with RMSE~23 W m^{-2} and BIAS close to 0.

Results shown above are relevant because Crawford and Duchon (1999) equation does not incorporate any coefficient in its expression, being able to ensure optimal results without previous calibration under different climate conditions. A physical explanation can be made from an analogy between effective emissivity of the atmosphere (ε_e) and effective emissivity of a heterogeneous and rough surface, ε_{sup}, as detailed below.

Valor and Caselles (1996) proposed a method to estimate ε_{sup} (thermal region; 8–14 μm) considering which represents the emissivity part corresponding to the radiation coming directly from the simple elements and is made up by the weighted sum of their emissivities. Another term (dε) called "cavity effect," which is related to the radiation that reaches the sensor indirectly by means of internal reflections, can be considered. Neglecting the cavity effect,

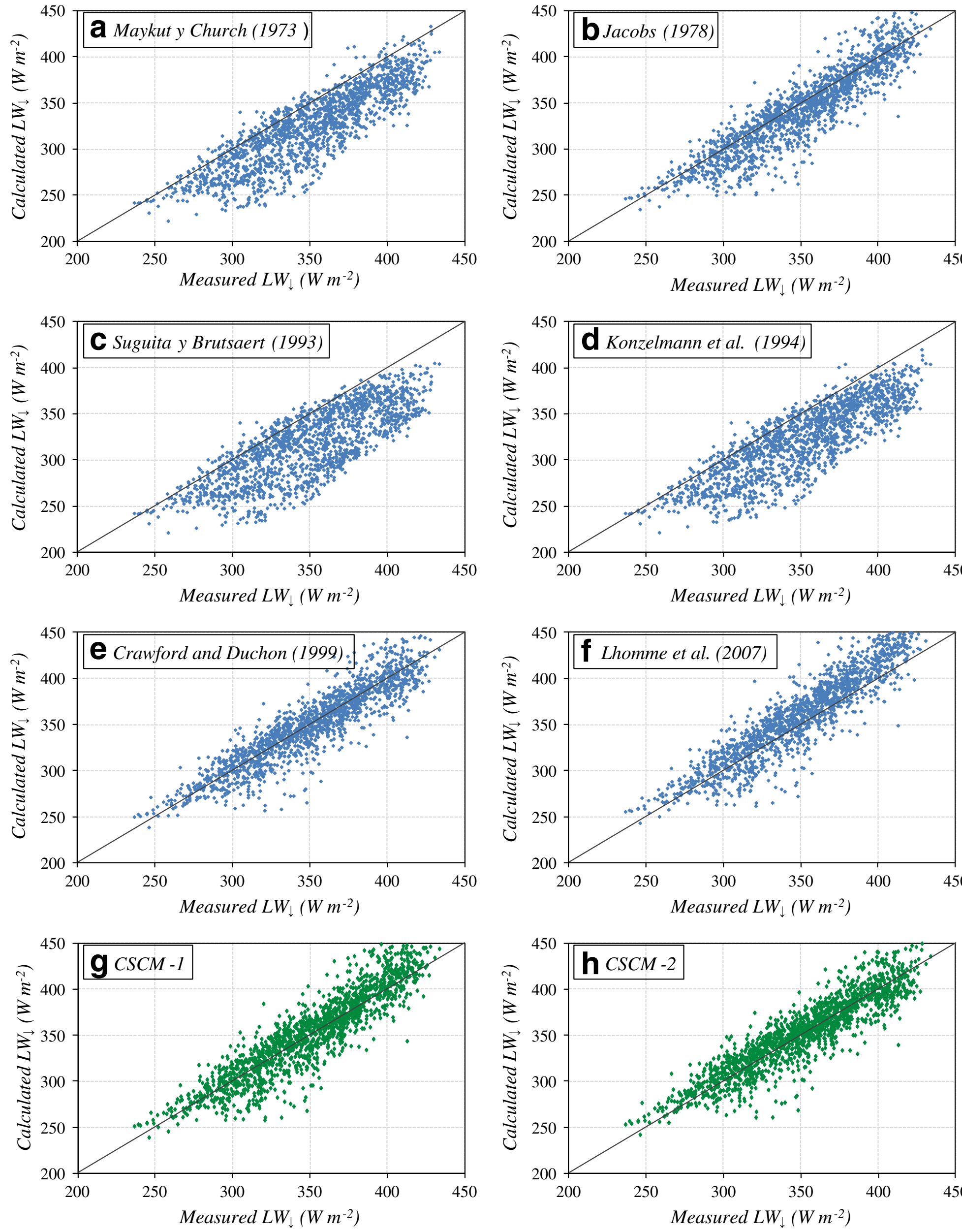

Fig. 5 Results of estimated downward longwave radiation versus measured values for the considered models under cloudy sky. The 1:1 line is shown on each plot

Table 8 Summary statistics of cloudy-sky correction models

CSCM	BIAS (W m^{-2})	RMSE (W m^{-2})	PRMSE (%)	*a* (W m^{-2})	*b*	r^2	*b**
Brutsaert (1975) without CSCM	−40	50	14	70±6	0.71±0.02	0.55	0.896±0.002
Maykut and Church (1973)	−26	30	9	14±5	0.89±0.01	0.74	0.926±0.001
Jacobs (1978)	−6	20	6	−14±4	1.02±0.01	0.83	0.982±0.001
Suguita and Brutsaert (1993)	−30	40	11	50±5	0.75±0.01	0.62	0.903±0.002
Konzelmann et al. (1994)	−30	40	11	30±5	0.83±0.01	0.71	0.917±0.002
Crawford and Duchon (1999)	−1	17	5	17±4	0.95±0.01	0.83	0.996±0.001
Lhomme et al. (2007)	12	24	7	−40±4	1.14±0.01	0.84	1.036±0.001
CSCM-1	2	19	5	−17±4	1.05±0.01	0.83	1.007±0.001
CSCM-2	2	17	5	28±4	0.92±0.01	0.83	1.003±0.001

simplified expression of Valor and Caselles (1996) equation is given by:

$$\varepsilon_{\text{sup}} = \varepsilon_v P_v + \varepsilon_g [1 - P_v] \quad (24)$$

where ε_v and ε_g are the emissivities of the vegetation and bare soil and P_v is the fractional vegetation cover which is obtained as function of the normalized difference vegetation index. The term $[1-P_v]$ is equal to soil proportion P_s. An analogy can be assumed to estimate the effective emissivity of the atmosphere, ε_e, considering a sky emissivity, ε_0, and a cloud emissivity, $\varepsilon_{\text{cloud}}$, with its proportions of sky P_{clear} and cloud P_{cloud}, respectively. Then, analogous expression is given by:

$$\varepsilon_e = \varepsilon_0 P_{\text{clear}} + \varepsilon_{\text{cloud}} [1 - P_{\text{clear}}] = \varepsilon_0 P_{\text{clear}} + \varepsilon_{\text{cloud}} P_{\text{cloud}} \quad (25)$$

When sky is completely overcast "emits as black body" and so $\varepsilon_e = \varepsilon_{\text{cloud}} = 1$, while that under clear-sky conditions $\varepsilon_e = \varepsilon_0$. P_{clear} and P_{cloud} can be written as:

$$P_{\text{clear}} = \frac{\text{SW}_\downarrow}{\text{SW}_{\downarrow 0}} = s \text{ and } \text{P}_{\text{cloud}} = \frac{\text{SW}_{\downarrow 0} - \text{SW}_\downarrow}{\text{SW}_{\downarrow 0}} = (1 - \text{s}) = \text{c}$$

substituting in Eq. (19) we obtain:

$$\varepsilon_e = \varepsilon_0 s + 1(1 - s) = \varepsilon_0 (1 - c) + c \quad (26)$$

that is similar to the equation proposed by Crawford and Duchon (1999). Besides having the best performance and simplest expression, this equation represents the emissivity parts corresponding to the radiation coming directly from the simple elements of clouds and clear sky and simply is made up by the weighted sum of their emissivities. Therefore, we choose the Crawford and Duchon (1999) equation to development of the MLRM-1 model in the next section of this study.

3.3 Proposed models

Here we presented the two new models developed to estimate downward longwave radiation. These models, MRLM-1 and MRLM-2 respectively, are given by:

$$\text{LW}_\downarrow = [\varepsilon_0(1 - \text{c}) + \text{c}]\sigma T_a^4 = [(-0.88 + 5.2 \times 10^{-3} T_a + 2.02 \times 10^{-3}\text{RH})(1 - \text{c}) + \text{c}]\sigma T_a^4 \quad (27)$$

and

$$\text{LW}_\downarrow = \varepsilon_e \sigma T_a^4 = [-0.34 + 3.36 \times 10^{-3} T_a + 1.94 \times 10^{-3}\text{RH} + 0.213\text{c}]\sigma T_a^4 \quad (28)$$

where T_a is expressed in kelvin, RH in percentage, and c varies between 0 and 1 (dimensionless). For first model, we used data of clear-sky conditions (N=2,258) to calibrate the coefficients within ε_0, being $\alpha_0=-0.88\pm0.04$, $\alpha_1=(5.2\pm0.1)\times 10^{-3}$ K^{-1} and $\alpha_2=(2.02\pm0.04)\times10^{-3}$ %$^{-1}$, while that for second model we used data both for clear and cloudy skies (N=5,595) to calibrate the coefficients within ε_e expression, being $\beta_0=-0.34\pm0.03$, $\beta_1=(3.36\pm0.09)\times10^{-3}$ K^{-1}, $\beta_2=(1.94\pm0.03)\times10^{-3}$ %$^{-1}$, and $\beta_3=0.213\pm0.002$.

Other measured data were used to validate the two proposed models. Results of the validations are shown in Fig. 6 and Table 9. We have considered three conditions to validate: (1) first with data under clear-sky conditions, (2) then we present the results for cloudy-sky conditions, and (3) finally for all sky conditions. Results of the best models previously tested are also show in Table 9.

From the results, we observe that two proposed models show good performances to estimate $\text{LW}_\downarrow$; RMSE values are between 12 and 16 W m^{-2}, without deviations and very good agreement with the measured data for the different sky conditions. Figure 6 shows higher $\text{LW}_\downarrow$ values for cloudy-sky conditions (red diamonds) due to increased effective emissivity of the atmosphere by clouds. Furthermore, these data show a largest scatter than those for clear-sky conditions (green triangles).

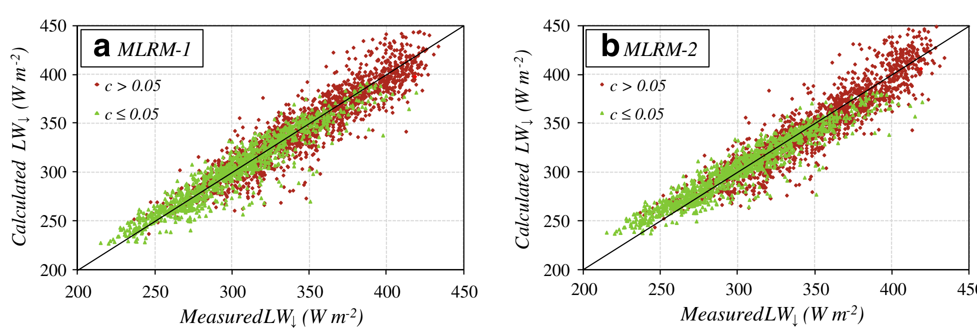

Fig. 6 Results of estimated downward longwave radiation versus measured values for the two proposed models. $c\leq0.05$ and $c>0.05$ represent clear and cloudy skies conditions, respectively. The 1:1 line is shown on each plot

Table 9 Summary statistics of two proposed models for different sky conditions

Sky conditions	Models	BIAS (W m^{-2})	RMSE (W m^{-2})	PRMSE (%)	a (W m^{-2})	b	r^2	b^*
(1) Clear-sky conditions	MLRM-1	1	12	4	26±3	0.92±0.01	0.90	1.001±0.001
$c \leq 0.05$	MLRM-2	2	13	4	60±2	0.81±0.01	0.90	1.004±0.001
N=1,185	Brutsaert (1975)	0	13	4	27±3	0.91±0.01	0.89	0.998±0.001
(2) Cloudy-sky conditions	MLRM-1	0	16	5	16±3	0.95±0.01	0.85	0.998±0.001
$c > 0.05$	MLRM-2	−1	16	5	11±3	0.97±0.01	0.86	0.996±0.001
N=1,613	Crawford-Duchon (1999) with Brutsaert (1975)	−1	17	5	17±4	0.95±0.01	0.83	0.996±0.001
(3) All sky conditions	MLRM-1	0	14	4	17±6	0.95±0.01	0.90	0.999±0.001
	MLRM-2	0	14	4	30±2	0.91±0.01	0.90	0.999±0.001
N=2,798	Crawford-Duchon (1999) with Brutsaert (1975)	0	15	5	18±2	0.95±0.01	0.89	0.998±0.001

In particular, for clear-sky conditions, MRLM-1 provides better results as compared to the MRLM-2 equation, with RMSE=12 W m^{-2}, lower a and b closer to 1. It is also noted that the results are slightly better than those obtained with the previously calibrated models. For cloudy-sky conditions, both proposed models present better performances which those previously calibrated models, where best agreements are observed with measured data ($r^2 \geq 0.85$). Finally, when data are considered under all sky conditions, the MLRM-1 shows a slightly better performance respect to MLRM-2 and also compared to rest of calibrated and tested models in this study.

4 Conclusions

In this paper, we present a detailed analysis to estimation of daytime downward longwave radiation using meteorological data measured in Tandil (Argentina). First, we tested six well-known simple models to estimate the $LW_{\downarrow}$ under clear conditions. Results showed that all models overestimate the measures of $LW_{\downarrow}$ considering daytime hourly data, where the best performances were obtained with the equations proposed by Brunt (1932) and Brutsaert (1975). In the second step, we applied a local fitting of coefficients to improve the performance of the models. Results showed no bias and were observed performance significant improvements by all clear-sky models considered. Models which do not employ the air relative humidity (Swinbank 1963; Idso and Jackson 1969) showed worse agreements with measured data (r^2=0.78) and larger errors with PRMSE equal to 6 %, while other models showed values of PRMSE=4 % and $r^2 \sim 0.90$. These results confirmed the importance of air relative humidity to estimate $LW_{\downarrow 0}$.

Moreover, we tested different equations to estimate $LW_{\downarrow}$ under cloudy-sky conditions. The best results were obtained with the equation proposed by Crawford and Duchon (1999), which showed no bias, RMSE equal to 17 W m^{-2}, and very good agreement with measured data (r^2=0.83). In this paper, we presented a physical explanation showing that this equation represents the radiation received at surface from the simple elements of the clouds and the clear sky as a weighted sum of their longwave radiation fluxes.

Finally, we presented two multiple linear regression models (MLRM-1 and MLRM-2) to estimate $LW_{\downarrow}$ at the surface for all sky conditions. Both new equations show better performance than the others models tested, with RMSE between 12 and 16 W m^{-2}, bias close to zero and best agreements with measured data ($r^2 \geq 0.85$). The results show the need to adjust the coefficients at local conditions to estimate the effective emissivity ε_0. We observed that to use a multiple linear regression model with air temperature and humidity and cloud fraction (within or without of the multiple linear regression) as input is a very good alternative operational to estimate $LW_{\downarrow}$.

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Appendix: Calculation of theoretical solar radiation

The theoretical solar radiation received in clear-sky conditions was estimated as product between extraterrestrial solar radiation, $SW_{\downarrow ex}$ (received at top of the atmosphere over a horizontal surface), and the transmissivity of the atmosphere, τ, through an empirical equation adapted from Beer's law:

$$SW_{\downarrow 0} = SW_{\downarrow ex}\,\tau = SW_{\downarrow ex} \exp\left(\frac{-0.018P}{(K_t \cos Z)}\right) \quad (A1)$$

where Z is the solar zenith angle, K_t is the turbidity coefficient (we considered clean air, being K_t=1), and P (hectopascal) is the atmospheric pressure calculated from altitude z (in meters) as:

$$P(z) = 1,013(1 - (0.0065z/293))^{5.26} \quad (A2)$$

and $SW_{\downarrow ex}$ is estimated as:

$$\begin{aligned} SW_{\downarrow ex} &= I_0 d_r^2 \cos Z \\ &= I_0 d_r^2 [\sin\varphi \sin\delta + \cos\varphi \cos\delta \cos H] \end{aligned} \quad (A3)$$

where I_0 is the solar constant (1,367 W m^{-2}) and d_r is the inverse of the Earth–Sun distance (in Astronomic units). φ, δ, and H are the latitude, the solar declination, and the hour angle, respectively. The variables d_r^2, δ, and H are calculated by means of the following expressions:

$$d_r^2 = 1 + 0.033 \cos\left(\frac{2\pi D}{365}\right) \quad (A4)$$

$$\delta = 0.409 \sin\left(\frac{2\pi D}{365} - 1.39\right) \quad (A5)$$

$$H = \left(\frac{\pi}{12}\right)(12 - t_s) \quad (A6)$$

being D the Julian day (1–365) and t_s (in hour) the local solar time calculated as:

$$t_s = t + L_c + S_c \quad (A7)$$

$$\begin{aligned} &S_c = 0.1645 \sin(2f) - 0.1255 \cos(f) - 0.0250 \sin(f) \\ &\text{with} \quad f = \frac{2\pi(D - 81)}{364} \end{aligned} \quad (A8)$$

where t (in hour) is the local time, L_c (in hour) is the correction for longitude, and S_c is the seasonal correction for the solar time (Allen et al. 1998; Lhomme et al. 2007).

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